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Representing Mathematical Models on the Web

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Part of DMSO Composability, (Doug Clark)

Why Ontologies?

- An Ontology provides a framework for metadata to describe models
- Meaningful interoperability requires machine readable metadata, expressing modeling concepts
  - Intelligent Agents need ontologically based metadata
- Example: HLA does not support metadata
  - HLA supports executable-to-executable communication
  - Limited semantics are expressed in the computer
  - Agreements and assumptions stored primarily in human memory, accessible only to a few

Benefits

- Open access to models by non-experts
  - Search and retrieval
- Begin automation of documentation
- Decision aids for simulation builders
  - Anticipate environmental effects in sim
- Animation engines and dead-reckoning
- Approach to true composability
  - Plug and play (not quite)

Tools

- Need to use Next Generation Language support
- There are tools that support math formalism
  - Mathematica, Maple, MuPad
  - Look for support for: objects; discrete methods
- OpenMath, MathML - Mathematical Markup Languages
- PyCES and PhysicsML
- DocBook, OMDoc

MOST OF THESE MODELS CAN BE USED IN "SCORE"

The "Zoo" of R&D Ocean Acoustic Models

ACOUSTIC WAVE EQUATION

HIGH FREQUENCY APPROXIMATION

RAY MODELS

- Gaussian Beams:
  - GRAB
  - HARPO ‡
  - ADAM
- GRASS

RANGE DEPENDENT

PARABOLIC EQUATION APPROXIMATION

- RAM
- EFEPE
- FEPE2WAY
- FEPE
- PORO-FEPE
- Split-Step
- UMPE / MMPE
- Navy Std PE 5.0 †
- PAREQ
- IFD PE
- DREP
- TDPE †
- NPE †
- FOR3D ‡
- 3DPE ‡
- FEPES §
- SHAPE §
- RAMS §

HYBRID MODELS

- PE-FFRAME
- WRAP §
- HARVEST § †

COUPLED MODES

- COUPLE
- KRAKEN
- SNAPC
- CENTRO/ANTS §

ADIABATIC MODE APPROXIMATION

- KRAKEN
- SNAP
- ORCA

DIRECT NUMERICAL SOLUTIONS

FINITE ELEMENTS

- SAFE §
- FFRAME
- FOAM
- FEMSOAP §

FINITE DIFFERENCE

- Stephen § †
- Levander § †
- FIDO †

RANGE INDEPENDENT

NORMAL MODES

- ORCA
- KRAKEN
- KRAKENC §
- SNAP
- NLNM
- AP2
- COMODE
- MODELAB

WAVENUMBER INTEGRATION

- OASES §
- SAFARI §
- PNSEB §
- FFP (Collins) §
- FFP (Kutchel) §

‡ 3-D MODELS
† TIME-DEPENDENT
§ SEISMO-ACOUSTIC (Shear Waves and Shear Attenuations)

A Physics-Based Model Ontology Layercake

• The thing modeled →	Physical object
• The Physics →	Physical concepts
• Language of physics →	Mathematical Expr.
• The easier solution →	Mathematical approx
• Let's get an answer! →	Discretized approx
• The next guy's grid →	Interpolation

Each layer to layer, downward transition is **informal**, **one-to-many**

Can we infer the Physical Concept from the last layer? **No!**

Multiple Efforts for Math Representations

- MIZAR - MIZAR language, document,
  - Dating to 1973
  - BNF specification, 2k definitions, 30k theorems
- CML
  - Compositional Modeling Language using EngMath (Engineering Math), in KIF-1990's
- OpenMath - an XML application
  - Dating to 1993
- MathML - an XML application
  - W3C Recommentation (The 1st app!)
- DocDook-MathML,OMDoc - XML applications

Web-Based = XML Applications

- XML is the emerging baseline for knowledge representation on the Web
- Content MathML and OpenMath are XML applications for specification of mathematical content
- DocBook-MathML and OMDoc are XML applications capable of representing mathematical documents

MathML

- Two Flavors specified
  - Presentation MathML and Content MathML
- Provides concept names for basic math
- Provides a construct for extension
- Many current web-browsers display it
- Reasonably mature (the first!) W3C Recommendation

Content MathML Elements and Attributes

- math /* root element */
- Presentation_tags, Spaces, Characters
- semantics, annotation , annotation-xml
- ci, cn, csymbol,
- apply, lambda, reln, fn,
- interval, list, matrix, matrixrow, set, vector, piecewise, piece, otherwise, lowlimit, uplimit, degree, logbase, domainofapplication, momentabout, condition,
- integers, rationals, reals, naturalnumbers, complexes, primes, exponentiale, imaginaryi, notanumber, true, false, pi, eulergamma, infinity, ne, eq, leq, lt, geq, gt,
- in, notin, notsubset, notprsubset, subset, prsubset, tendsto,, inverse, ident, domain, codomain, image, fn, compose, root, abs, conjugate, factorial, minus, arg, real, imaginary, floor, ceiling, quotient, divide, minus, power, rem, plus, times, max, min, gcd, lcm,
- int, diff, partialdiff, divergence, grad, curl, laplacian, sum, product, limit,
- exp, ln, log, sin, cos, tan, sec, csc, cot, sinh, cosh, tanh, sech, csch, coth, arcsin, arccos, arctan,
- moment, mean, sdev, variance, median, mode,
- determinant, transpose, vectorproduct, scalarproduct, outerproduct, selector,
- not, implies, equivalent, approx, factorof, and, or, xor, forall, exists, card, setdiff, union, intersect, cartesianproduct,
- sep,
- declare , constructor
- bvar

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Presentation vs. Content: Example

- What does a superscript mean?

$$x^i = \pi$$

- Exponent, label, element? What content?

```
<apply>
  <eq/>
    <apply>
      <power/>
      <ci>x</ci>
      <ci>i</ci>
    </apply>
  <cn type="constant">&pi;</cn>
</apply>
```

```
<apply>
  <eq/>
    <apply>
      <selector/>
        <ci type="vector">x</ci>
        <ci>i</ci>
    </apply>
  <cn type="constant">&pi;</cn>
</apply>
```

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Differential Equations

$$\nabla^2 G - \frac{\nabla \rho}{\rho} \bullet \nabla G + \frac{1}{c^2}\frac{\partial^2 G}{\partial t^2} = -\delta(r - r')\delta(t - t')$$

```
<apply>
   <divergence/>
   <apply>
      <gradient/>
      <ci type=”function”>G</ci>
   </apply>
</apply>
```

```
<apply>
   <scalarproduct/>
      <apply>
         <divide/>
         <apply>
            <gradient/>
            <ci type=”function”>&rho</ci>
         </apply>
         <ci>&rho</ci>
      </apply>
   <apply>
      <gradient/>
      <ci type=”function”>G</ci>
   </apply
</apply>
```

```
<apply>
   <multiply/>
   <apply>
      <power/> <ci type=”function”>c</ci>
      <cn>-2</cn>
   </apply>
   <apply>
      <partialdiff/>
      <bvar><degree><cn>2</cn></degree>
         <ci>t</ci>
      </bvar>
      <degree><cn>2</cn></degree>
      <ci type=”function”>G</ci>
   </apply>
</apply>
```

MathML Shortcomings

- Last example - no Dirac delta function
- Grad, Div, Curl are inherently Euclidean
  - Require metric space, tensor descriptions
- Private extensions receive little scrutiny
- "Standard" is not easily extended

- In summary - weak extensibility when a lot of extensibility is required

OpenMath

- Exclusively content representation
- “Interoperable” with MathML
  - OpenMath Society close with W3C
- Basic specification is core of OpenMath
- Content Dictionaries (CDs) provide terms
  - Website repository for candidate CDs
- Emphasis on properties

OpenMath is Also Necessary

- Content representation and *properties*
- Mutual agreement on complementarity
- Interoperates with MathML
- XSLT translation to MathML provided
- Excellent extensibility
  - CDs on public view
  - Process for approval of CDs
- OWL translations exist for many CDs

Mathematical Documents

- Scope of declarations confined to a math element
- Symbol with a given identity may have properties that evolve in a document
- Need to state systems of equations as models
- Display is less important than meaning

Documentation Requirements

- Presentation
- Document Structure (Dublin Core)
- Data Structure, Semantics
  - Checking consistency, validation
  - Exchange with other applications
  - Reuse, draw inference

Documentation Tools

- DocBook MathML
  - DocBook is a W3C Recommendation
  - DocBook MathML is an extension
- OMDoc built on MathML and OpenMath
  - Based on “modules” of elements: DOC, Dublin Core, Creative Commons, Content MathML, OpenMath, Math Text, Mathematical Statements, Semantic Reference, Abstract Data Types, Proofs, Complex Theories, …

Summary and Direction

- MathML, OpenMath
  - Basic foundation for web-based approach
  - OpenMath is not W3C
- OMDoc appears very promising
  - Principals receptive to collaboration
  - Does it satisfy requirement of establishing mappings between layers of abstraction?
  - Does it provide adequate scoping?
  - OMDoc is not W3C